D-branes as black holes

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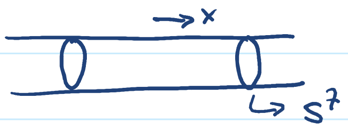
We can look for solutions of the supergravity theory that are black hole-like.

Eg The Schwarzschild solution in 10D

$$ds^2 = -\left(1 - \frac{\mu}{r^7}\right) dt^2 + \frac{dr^2}{1 - \mu/r^7} + r^2 d\Omega_8 \qquad \mu \propto GM$$

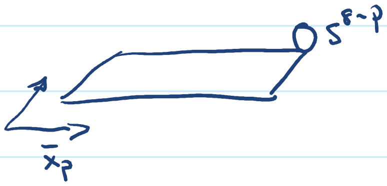
or black strings

$$ds^2 = -\left(1 - \frac{\mu}{r^6}\right) dt^2 + dx^2 + \frac{dr^2}{1 - \mu/r^6} + r^2 d\Omega_7$$



or black p-branes

$$ds^2 = -\left(1 - \frac{\mu}{r^{7-p}}\right) dt^2 + d\vec{x}_p^2 + \frac{dr^2}{1 - \frac{\mu}{r^{7-p}}} + r^2 d\Omega_{8-p}$$



These solutions are neutral, but we'll be interested in solutions that are charged under one (or more) of the p-form fields.

Since particles (0-branes) couple $\overset{\text{source}}{\checkmark}$ electrically to 1-form potentials A_μ

$$F_{\mu\nu} = 2\partial_{[\mu} A_{\nu]} \qquad F_{(2)} = dA_{(1)}$$

strings (1-branes) couple to 2-form $B_{\mu\nu}$

$$H_{(3)} = dB_{(2)}$$

p-branes couple to (p+1)-form $A_{\mu_0 \cdots \mu_p}$

$$F_{(p+2)} = dA_{(p+1)}$$

we expect to find solutions that are extended along p spatial directions sourcing $A_{(p+1)}$

There will be analogues of a solution for an "electron" that sources an electric field and a gravitational field: Reissner-Nordstrom w/ $Q > M$

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r} + \frac{Q^2}{r^2}} + r^2 d\Omega_2$$

$$A_t = -\frac{Q}{r}$$

This is singular, but we expect that QED effects resolve the classical electric field into a quantum electric field and smooths out the singularity (no quantum gravity)

For instance, we can find a solution with an electric $B_{(2)}$ field, ie B_{tx}, which describes a string-like object extended along x:

<u>F-string</u>

$$ds^2 = \frac{1}{h}\left(-dt^2 + dx^2\right) + dr^2 + r^2 d\Omega_7$$

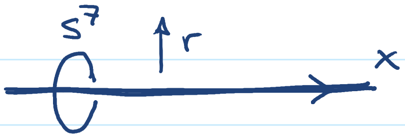
c_1 = number

$$h = 1 + \frac{c_1 g_s^2 N \ell_s^6}{r^6} \equiv 1 + \frac{q^6}{r^6} \qquad q = \left(c_1 g_s^2 N\right)^{1/6} \ell_s$$

$$B_{tx} = 1 - \frac{1}{h}$$

$$e^{2\Phi} = 1/h \quad \left(r \to 0 \Rightarrow e^{\Phi} \to 0 : \text{weak coupling} \right)$$

Solution to NS-NS sector, common to all closed strings



$N = \#$ of fundamental strings

The F-string is extremal, ie "$Q = M$" $(\sim g^2 N_1)$
but it doesn't have a regular horizon.
There exists a non-extremal version w/ horizon
(black F-string)

$$ds^2 = \frac{1}{h(r)}\left(-f(r)\,dt^2 + dx^2\right) + \frac{dr^2}{f(r)} + r^2\, d\Omega_7$$

$$f(r) = 1 - \left(\frac{r_0}{r}\right)^6 \qquad \text{horizon at } r = r_0$$

This solution could be boosted along x, to give it momentum.

$$t \to \cosh\eta\, t + \sinh\eta\, x \qquad \eta = \text{rapidity}$$
$$x \to \sinh t + \cosh x$$

It becomes a F_1-P string

D3-brane

$$ds^2 = h^{-1/2}(r)\left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + h^{1/2}(r)\left(dr^2 + r^2\, d\Omega_5\right)$$

$$h = 1 + \frac{4\pi g_s N \ell_s^4}{r^4} = 1 + \frac{q^4}{r^4} \qquad q = (4\pi g_s N)^{1/4}\, \ell_s$$

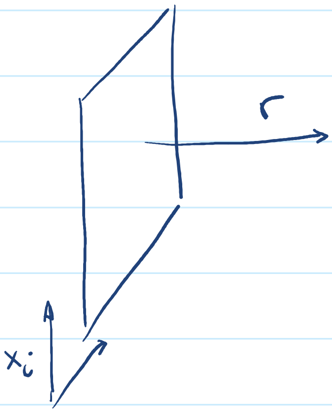
$$F_{(5)} = (1+*)\, dt \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dh^{-1}$$

$$e^{2\phi} = \text{constant} = g_s^2$$

The worldvolume is (t, x_1, x_2, x_3)

Transverse : r, Ω_5

The solution is AF in Transverse directions
IT doesn't have a curvature



singularity at $r=0$.
We'll see it's a horizon
but The Temperature and
The area are zero.

There exists a version with $T_H \neq 0$ and $A_H \neq 0$

Black D3-brane :

$$ds^2 = h^{-1/2}(r)\left(-f(r)\,dt^2 + d\bar{x}_3^2\right) + h^{1/2}(r)\left(\frac{dr^2}{f(r)} + r^2 d\Omega_5\right)$$

with $h(r) = 1 + \frac{q^4}{r^4}$ $\qquad f(r) = 1 - \left(\frac{r_0}{r}\right)^4$

Horizon at $r = r_0$

$$A_H = h^{1/2}(r_0)\, r_0^5\, \Omega_5$$