AdS_2,3 from near-horizon limits

martes, 6 de diciembre de 2022 0:13

AdS_2 from near-horizon (near-) extremal Reissner-Nordström

$$ds^2 = -\left(1-\frac{2m}{r}+\frac{q^2}{r^2}\right)dt^2 + \frac{dr^2}{1-\frac{2m}{r}+\frac{q^2}{r^2}} + r^2 d\Omega_2$$

Outer event horizon at $r = r_+ = m + \sqrt{m^2-q^2}$ $\qquad m \geq q$

Change $r \to r+q$, define $r_0 = r_+ - q$
Then $\qquad = m - q + \sqrt{m^2-q^2}$

$$ds^2 = -\frac{f(r)}{\left(1+\frac{q}{r}\right)^2}dt^2 + \left(1+\frac{q}{r}\right)^2\left(\frac{dr^2}{f(r)} + r^2 d\Omega_2\right)$$

$$f(r) = 1 - \frac{r_0}{r}$$

Charge $\propto q$ $\qquad r_0 =$ non-extremality parameter

Horizon at $r = r_0$. Area $= 4\pi(r_0+q)^2$

Extremal limit: $r_0 = 0$, horizon at $r=0$, finite area

In this case taking $r \ll q$, near-horizon limit:

$$ds^2 \simeq -\frac{r^2}{q^2}dt^2 + q^2\frac{dr^2}{r^2} + q^2 d\Omega_2 \qquad r = \frac{q^2}{z}$$

$$= q^2\left(\frac{-dt^2+dz^2}{z^2} + d\Omega_2\right)$$

<u>Poincaré-AdS_2</u> $\times S^2$

Now consider near-extremal, near-horizon

Take $r, r_0 \ll q$

$$ds^2 = -\frac{r(r-r_0)}{q^2}dt^2 + q^2\frac{dr^2}{r(r-r_0)} + q^2 d\Omega_2$$

change $r = \frac{r_0}{2}(\tilde{r}+1)$ $\quad t = \frac{2g^2}{r_0}\tilde{t}$. Then

$$ds^2 = g^2\left(-(\tilde{r}^2-1)\,d\tilde{t}^2 + \frac{d\tilde{r}^2}{\tilde{r}^2-1} + d\Omega_2\right)$$

$\underline{\text{Rindler-}AdS_2} \times S^2$

To find global $AdS_2 \times S^2$ we must start from a 4D traversable wormhole

Now we obtain AdS_3 and BTZ from a charged black string in 6D (could also do in 5D)

Black D1-D5 string:

$$ds^2 = \frac{1}{\sqrt{h_1 h_5}}\left(-f(r)\,dt^2 + dx^2\right) + \sqrt{h_1 h_5}\left(\frac{dr^2}{f(r)} + r^2 d\Omega_3\right)$$

$$h_{1,5} = 1 + \frac{r_{1,5}^2}{r^2} \qquad f(r) = 1 - \left(\frac{r_0}{r}\right)^2$$

x: direction along string, $x \sim x + L$

r_1, r_5: $\propto$ D1, D5-brane charges

r_0: non-extremality parameter

[Might consider boosting it along x to get D1-D5-P

$$ds^2 = \frac{1}{\sqrt{h_1 h_5}}\left(-dt^2 + dx^2 + \frac{r_0^2}{r^2}\left(\cosh\alpha\, dt + \sinh\alpha\, dx\right)^2\right)$$
$$+ \sqrt{h_1 h_5}\left(\frac{dr^2}{f(r)} + r^2 d\Omega_3\right)$$

α: boost parameter along x]

Horizon at $r = r_0$

Near-extremal: $r, r_0 \ll r_1, r_5$ $\qquad h_{1,5} \simeq \frac{r_{1,5}^2}{r^2}$

Set $\alpha = 0$ for simplicity

$$ds^2 \simeq \frac{r^2}{r_1 r_5}\left(-\left(1 - \frac{r_0^2}{r^2}\right)dt^2 + dx^2\right) + r_1 r_5 \frac{dr^2}{r^2 - r_0^2} + r_1 r_5\, d\Omega_3$$

rescale $(t, x) = r_1 r_5 (\tau, \phi)$ $\qquad \Delta\phi = \frac{L_x}{r_1 r_5}$

$$ds^2 = r_1 r_5 \Big[\underbrace{-(r^2 - r_0^2)\, d\tau^2 + \frac{dr^2}{r^2 - r_0^2} + r^2 d\phi^2}_{\text{static BTZ}} + \underbrace{d\Omega_3}_{\times\ S^3}\Big]$$

If we have $\alpha \neq 0$ then we get rotating BTZ $\times S^3$

If $\alpha = 0$ $r_0 = 0$ we get Poincaré-$AdS_3 \times S^3$

To get Global $AdS_3 \times S^3$ we must start from a "supertube"

Observe that energy of black string above extremality $\propto r_0^2 \sim$ BTZ mass momentum of black string $\propto$ BTZ ang mom

entropy of black string $\propto$ entropy of BTZ $\times$ area of S^3

In string theory, there is a CFT_2 for the D1-D5 system, which is used to study the microscopic properties of the BTZ black hole.

The CFT in the "free" limit is a non-linear σ-model on the symmetric orbifold over $N_1 N_5$ copies of T^4 or K^3